STUDY GUIDE FOR THE FINAL

PEYAM RYAN TABRIZIAN

Note: The final will cover everything from section 1.1 to section 6.3 inclusive, **EXCEPT** sections 1.4, 2.1, 3.11, 4.6, and 4.8.

Note: Study tip: Start by studying Chapters 5 and 6, because the final will emphasize those chapters. Then study Chapters 2, 3, and 4. Don't waste your time and energy on Chapter 1, because it's really only meant to be a warm-up for the semester (maybe briefly take a look at sections 1.3 and 1.6). Also, for Chapter 3, start by studying section 3.5, because the previous sections you should already know!

Note: 1.5.4 means Problem 4 in section 1.5 !

- Know how to:

1. Chapter 1: Functions and Models

- Find the domain and the range of a function, given a graph or a formula (1.1.5, 1.1.30, 1.1.32)
- Graph new functions from old ones (1.3.13, 1.3.15)
- Determine if a function is one-to-one, given a graph or a formula (1.6.5, 1.6.9, 1.6.11)
- Find a formula for the inverse of a function (1.6.21, 1.6.23, 1.6.26)
- Solve equations involving ln (1.6.49, 1.6.50)
- Prove formula for functions involving inverse trig functions, using the triangle method (1.6.64, 1.6.65, 1.6.67)

Date: Monday, May 2nd, 2010.

2. Chapter 2: Limits and Derivatives

- Given a graph, find a given limit if it exists or explain why it does not exist (2.2.6)
- **Find limits of a function:**
  - Step 1: Just by plugging in (2.3.3)
  - Step 2: By noticing that it's of the form $\frac{1}{0^+} = \infty$ or $\frac{1}{0^-} = -\infty$ (2.2.25, 2.2.29,)
  - Step 3: By factoring out the numerator and the denominator and simplifying (2.3.13, 2.3.17, 2.3.18, 2.3.26)
  - Step 4: Whenever there is a square root, by multiplying numerator and denominator by the conjugate form (2.3.21, 2.3.23, 2.3.29, 2.3.30)
  - Step 5: By using the squeeze theorem (2.3.37, 2.3.38)
  - Step 6: By calculating $\lim_{x\to a^-}$ and $\lim_{x\to a^+}$ and by noticing that they're equal or not (2.3.39, 2.3.42, 2.3.47)
  - Step 7: By using l'Hopital's rule (see Chapter 4)

  Note: If you need more practice, try the following set of problems: 2.3.22, 2.3.8, 2.3.11, 2.3.12, 2.3.36
- Find limits using the $\epsilon - \delta$ notion of a limit (2.4.19, 2.4.25, 2.4.29, 2.4.30, 2.4.31, 2.4.36) (don't focus **too** much on this! The final covers other, more important topics as well!
- Given a graph, state at which points a function is continuous (2.5.3)
- Evaluate limits using continuity (2.5.31, 2.5.34, 2.6.34)
- Find numbers at which a function is discontinuous (2.5.37, 2.5.39)
- Show that an equation has at least one solution, using the Intermediate Value Theorem (2.5.47, 2.5.49, 2.5.61, 2.5.65)
- **Find limits at infinity of a function:**
  - Step 1: Just by plugging in (2.6.15)
  - Step 2: By factoring out the highest power out of an expression (2.6.31, 2.6.50)
  - Step 3: By factoring out the highest power of the numerator and the denominator (2.6.17, 2.6.19, 2.6.21, 2.6.33)
  - Step 4: By factoring out the highest power of x out of a square root (2.6.22, 2.6.23, 2.6.24)
  - Step 5: By using the conjugate form, making sure to do Step 4 first (2.6.25, 2.6.26, 2.6.27)
  - Step 6: By using the squeeze theorem (2.6.35, 2.6.57)
  - Step 7: By using l'Hopital's rule (see Chapter 4)

  Note: If you need more practice, try the following set of problems: 2.6.30, 2.6.29, 2.6.20, 2.6.37, 2.6.44
- Find the equation of the tangent line to a function at a point (2.7.5, 2.7.7)
- Find the derivative of a function at a given point or in general, using the definition of the derivative (2.7.25, 2.7.30, 2.8.19, 2.8.21, 2.8.24)
- Recognize a limit as the derivative of a function (2.7.31, 2.7.45)

3. CHAPTER 3: DIFFERENTIATION RULES

- Find the derivative of a function, using the power rule, the product/quotient rules or the chain rule (3.1.13, 3.2.15, 3.2.18, 3.3.6, 3.3.9, 3.4.3, 3.4.39, 3.4.45, 3.6.5, 3.6.19, 3.6.29, 3.6.45, 3.6.49)
- Find the derivative of a function using implicit differentiation (3.5.5, 3.5.17, 3.5.19)
- Find the derivative of a function of the form $f(x)^{g(x)}$ using logarithmic differentiation (3.6.39, 3.6.41, 3.6.43, 3.6.50)
- Find the equation to the tangent line to a given curve at a given point (3.4.53, 3.5.27, 3.5.29, **3.5.40, 3.5.41, 3.5.42**)
- Find limits involving $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$ and $\lim_{x\to 0} \frac{\cos(x)-1}{x} = 0$ (3.3.49, 3.3.50, 3.3.42)
- Given the position function of a particle, find its velocity and acceleration, and determine things such as the distance traveled, when it is speeding up etc. (3.7.1, 3.7.3)
- Know the solution to $y' = ky$, if in doubt, Ce^{kt} out! :), and solve problems involving exponential growth and decay (3.8.3, 3.8.9, 3.8.11)
- **Solve related rates problems** (For example, try out 3.9.6, 3.9.15, 3.9.17, 3.9.24, 3.9.30, 3.9.36, 3.9.38, 3.9.43, **3.9.44**)
- Find the linear approximation of a function at a given point (3.10.1, 3.10.4)
- Find the differential dy of a function (3.10.15)
- Use a linear approximation or differentials to estimate a given number (3.10.23, 3.10.25, 3.10.28)
- Use differentials to estimate the maximum and the relative error (3.10.34, 3.10.35, 3.10.40)

4. Chapter 4: Applications of Differentiation

- Given a graph, determine all the absolute and local maximums and minimums of a function (4.1.5, 4.1.6)
- Find the critical numbers of a function (4.1.29, 4.1.35, 4.1.39, 4.1.42)
- Find the absolute maximum and minimum values of a function on a given interval (4.1.47, 4.1.51, 4.1.53, 4.1.57, 4.1.60, 4.1.61, 4.1.63)
- Using Rolle's theorem (and the Intermediate Value Theorem), show that an equation has exactly one solution, or at most one solution, etc. (4.2.17, 4.2.18, 4.2.19, 4.2.22)
- Solve problems using the Mean Value Theorem (4.2.23, 4.2.24, 4.2.25, 4.2.27, 4.2.29, 4.2.34, 4.2.35, 4.2.36)
- Show that an identity holds by differentiating it, and evaluating it at one point (4.2.32, 4.2.33)
- Graph a function, given information about its derivative and second derivatives (4.3.25, 4.3.27)
- **Note:** Section 4.3 is not that important, because it is basically a prerequisite to section 4.5, so if you master section 4.5, then you also master section 4.3.
- Evaluate limits using l'Hopital's rule (any problem in section 4.4 works, try out 4.4.11, 4.4.13, 4.4.17, 4.4.21, 4.4.27, 4.4.31, 4.4.39, 4.4.43, 4.4.49, 4.4.51). Always remember to check the indeterminate form first, and see if there is an easier way to solve the problem!
- Use l'Hopital's rule to evaluate indeterminate powers $0^0, \infty^0, 1^\infty$ (4.4.53, 4.4.59, 4.4.60)
- **Use the DISAIC method to sketch the graph of a function** (4.5.13, 4.5.25, 4.5.31, 4.5.48, 4.5.52, 4.5.56, also $y = e^{\frac{1}{x}}$ is super popular), don't worry about slant asymptotes, and remember that you can always check your answer with a calculator!
- **Solve opimization problems** (any problem in section 4.7 would do, try out 4.7.13, 4.7.19, 4.7.27, 4.7.37, 4.7.39, 4.7.46, 4.7.52, 4.7.69, 4.7.72)
- Don't waste your time on section 4.9, it's just a warm-up for chapter 5.

5. CHAPTER 5: INTEGRATION

- Estimate the integral of a function, using right endpoints, left endpoints, or midpoints, **given n** (5.1.3, 5.1.5, 5.2.1, 5.2.2, 5.2.3)
- Express the area under the graph of a given function as a limit (5.1.17, 5.1.18, 5.1.19)
- Determine a region whose area is equal to a given limit (5.1.20, 5.1.21)
- Express a limit as a definite integral on a given interval (5.2.17, 5.2.19, 5.2.20)
- **Evaluate integrals directly, using the definition of an integral** (5.2.21, 5.2.22, 5.2.23, 5.2.24, 5.2.25)
- Evaluate integrals by interpreting them in terms of areas (5.2.34, 5.2.36, 5.2.37, 5.2.39)
- Use the comparison property of integrals to prove an identity involving integrals (5.2.52, 5.2.54, 5.2.61, 5.2.62)
- Express a limit as a definite integral and evaluate that integral (5.2.69, 5.2.70, 5.3.65, 5.3.66)
- **Use the FTC Part I to find the derivative of a function** (5.3.7, 5.3.9, 5.3.13, 5.3.17, 5.3.18, 5.3.54, 5.3.55, 5.4.68)
- **Use the FTC Part II to evaluate integrals** (any problem between 5.3.19 and 5.3.42, as well as between 5.4.21 and 5.4.44, would do, try out 5.3.25, 5.3.31, 5.3.37, 5.3.38, 5.3.40 for example)
- Find indefinite integrals/antiderivatives (5.4.9, 5.4.16, 5.4.17, 5.4.18)
- Interpret what definite integrals represent in real life (5.4.51, 5.4.52)
- Evaluate integrals by using the substitution rule (any problem between 5.5.7 and 5.5.46, as well as between 5.5.51 and 5.5.70 would do, try out 5.5.7, 5.5.19, 5.5.21, 5.5.23, 5.5.25, 5.5.39, 5.5.54, 5.5.59, 5.5.67, 5.5.68 for example)
- Remember that if f is odd, then $\int_{-a}^{a} f(x)dx = 0$ (5.5.57, 5.5.73)

6. Chapter 6: Applications of integration

- Find the area of the region enclosed by the given curves (6.1.8, 6.1.9, 6.1.13, 6.1.17, 6.1.19, 6.1.21, 6.1.40)
- Evaluate an integral involving absolute values (6.1.31, 6.1.32)
- Find a number such that a given line divides a region into two regions with equal area (6.1.49, 6.1.50, 6.1.51)
- Find a volume using the disk or washer method (6.2.3, 6.2.6, 6.2.9, 6.2.11, 6.2.16, 6.2.17)
- Find volumes of more sophisticated solids using the disk or washer method (6.2.49, 6.2.50, 6.2.51, 6.2.63, 6.2.67, 6.2.70)
- Find a volume using the **definition** of the volume, i.e. find $A(x)$ explicitly (6.2.52, 6.2.54, 6.2.56, 6.2.57, 6.2.58)
- Find the volume of a solid using cylindrical shells (6.3.3, 6.3.5, 6.3.9, 6.3.11, 6.3.13, 6.3.15, 6.3.19)
- Find the volume of more sophisticated solids using the shell method (6.3.43, 6.3.44, 6.3.45, 6.3.46)
- Recognize an integral as the volume of a solid (6.3.29, 6.3.31)
- As a grand finale, evaluate the volume of a solid using any of the 3 methods taught in this chapter (6.3.37, 6.3.38, 6.3.39, 6.3.40, 6.3.41, 6.3.42)